MUTUAL IMPEDANCE BETWEEN TWO SKEW ANTENNAS

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MUTUAL IMPEDANCE BETWEEN TWO SKEW ANTENNAS

by

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by

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PREFACE

In this paper a general formula derived by Professor J. G. Chaney of the U. S. Naval Postgraduate School for the mutual impedance between two skew antennas of different lengths is reduced to the specific case of two coplanar, half-wave elements with half-wave center to center spacing and the impedance for various angles of skew computed. The results of the measurements on the slot complement of this antenna system are compared with the calculated values. Additional methods of performing this type measurement are discussed.

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The author wishes to thank Dr. John Taylor, under whose direction the experimental work for this paper was carried out, and Dr. J. T. Bolljahn for their many helpful suggestions. Also, the author is indebted to Professor J. G. Chaney for suggesting the topic of this paper and for the use of his original notes.

This investigation was undertaken by Lt. N. W. Gokey, who was temporarily assigned to Stanford Research Institute from the U. S. Naval Postgraduate School at Monterey, in connection with the Naval Industrial Experience Program. The problem was suggested by Prof. J. G. Chaney of the U. S. Naval Postgraduate School, and was an outgrowth of work being done under BuShips Project Order No. 10770/55 to Naval Postgraduate School, Monterey.

ABSTRACT

In this report a general formula derived by J. G. Chaney of the U. S. Naval Postgraduate School at Monterey, California, for the mutual impedance between two skew (non-parallel) antennas of different lengths is reduced to the specific case of two coplanar, half-wavelength elements with half-wavelength center-to-center spacing. The impedance for various angles of skew is computed. The results of the measurements made on the slot complement of this antenna system are compared with the calculated values. There is good correlation between the shapes of the curves for measured and theoretical results, although the measured results averaged 30% above the values predicted by the theory.

Other techniques for finding the mutual impedance between two antennas are discussed; some of these may present fewer difficulties.

CONTENTS

ILLUSTRATIONS

TABLES

MUTUAL IMPEDANCE BETWEEN TWO SKEW ANTENNAS

CHAPTER 1

INTRODUCTION

In a recent unpublished paper[1*] Chaney derived an expression for the mutual impedance between two center-driven dipole antennas arbitrarily oriented with respect to each other. He approaches the problem by means of his generalized circuit theory concept,[2] and expresses the mutual impedance in terms of associated sine and cosine integral functions. These functions were introduced by Chaney in his studies of rhombic antennas[3] and a limited table of them is available.[4] More extensive tables[5] are in the process of publication.

In the present investigation, which was suggested by Professor Chaney, the general formula for the mutual impedance between two antennas has been reduced to the specific case of two coplanar, half-wavelength elements with half-wavelength center-to-center spacing, and the impedance for various angles of skew has been computed. Measurements have been made for this specific case and the results compared with the calculated values.

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* References are listed at the end of the report.

CHAPTER 2

SKEW WIRE ANTENNAS

A. DERIVATION OF MUTUAL IMPEDANCE RELATION

Chaney's generalized circuit theory concept starts with Maxwell's equations, from which an expression is derived for the complex power associated with a circuit. This power is separated into a complex input power and a complex power (including the radiated power) into the external fields associated with the circuit. The internal and external impedances are obtained from these powers without assuming specific current distributions. However, for computing mutual impedance values, a current distribution must be assumed.

The mutual impedance between two antenna configurations (see ref. 2) is given in the generalized circuit by

$$Z_{12} = \frac{j30}{k} \oint_1 \oint_2 R_e \left[f_1 \ (P_1)^* \ f_2 \ (P_2) \right] \ \Diamond_1 \left[e(r_{12}) \ d\,\bar{r}_2 \right] \cdot d\,\bar{r}_1 \qquad (1)$$

where

P_1 = any point along the axis of wire No. 1,

P_2 = any point along the axis of wire No. 2,

r_{12} = the distance from P_1 to P_2,

$e(r_{12}) = e^{-jkr_{12}}/r_{12}$,

$k = \omega\,(\mu_0 \epsilon_0)^{\frac{1}{2}} = 2\pi/\lambda$,

$\mu_0 = 4\pi \times 10^{-7}$ henries per meter,

$\epsilon_0 = (36\pi \times 10^{9})^{-1}$ farads per meter,

$\Diamond_1 = \nabla_1 \ (\nabla_1 \cdot \ ) + k^2$ which is the operator "deltil," with the subscript indicating the position at which the differentiations are to be performed,

$f_1(P_1)$ = the current distribution function along wire No. 1, and

$f_2(P_2)$ = the current distribution function along wire No. 2.

Complex conjugate quantities are indicated by a star (*), while R_e stands for the real part of the associated expression. Postulating sinusoidal current distributions,

$$I_1 = I_{01} \sin k|l_1 - s_1|$$

and (2)

$$I_2 = I_{02} \sin k|l_2 - s_2|$$

on the two antennas, their mutual impedance is given by:

$$Z_{12} = \frac{j30}{k} \oint_1 \oint_2 \sin k|l_1 - s_1| \sin k|l_2 - s_2| \Diamond_1 [e(r_{12}) d\bar{r}_2] \cdot d\bar{r}_1 \tag{3}$$

$$\begin{aligned}
\frac{jk}{30} Z_{12} = & \int_{-l_1}^{0} \int_{-l_2}^{0} \sin k(l_1 + s_1) \sin k(l_2 + s_2) \left[\frac{\partial^2}{\partial s_1 \partial s_2} - k^2 \cos 2\psi \right] e(r_{12}) ds_2 ds_1 \\
& + \int_{-l_1}^{0} \int_{0}^{l_2} \sin k(l_1 + s_1) \sin k(l_2 - s_2) \left[\frac{\partial^2}{\partial s_1 \partial s_2} - k^2 \cos 2\psi \right] e(r_{12}) ds_2 ds_1 \\
& + \int_{0}^{l_1} \int_{-l_2}^{0} \sin k(l_1 - s_1) \sin k(l_2 + s_2) \left[\frac{\partial^2}{\partial s_1 \partial s_2} - k^2 \cos 2\psi \right] e(r_{12}) ds_2 ds_1 \\
& + \int_{0}^{l_1} \int_{0}^{l_2} \sin k(l_1 - s_1) \sin k(l_2 - s_2) \left[\frac{\partial^2}{\partial s_1 \partial s_2} - k^2 \cos 2\psi \right] e(r_{12}) ds_2 ds_1
\end{aligned} \tag{4}$$

Referring to Fig. 1

$$r_{12} = \left\{ [s_1 - x_0 \cos \phi - s_2 \cos 2\psi]^2 + [x_0 \sin \varphi + s_2 \sin 2\psi]^2 + h^2 \right\}^{\frac{1}{2}} \tag{5}$$

where

h = the shortest distance between the lines of the two antenna axes,

x_0 = the length of the projection of the line joining the centers of the antennas upon the plane through antenna 1 parallel to antenna 2.

φ = the angle between x_0 and the positive sense of antenna 1, and

2ψ = the vertex angle between positive senses of antenna 1 and the coplanar line parallel with antenna 2.

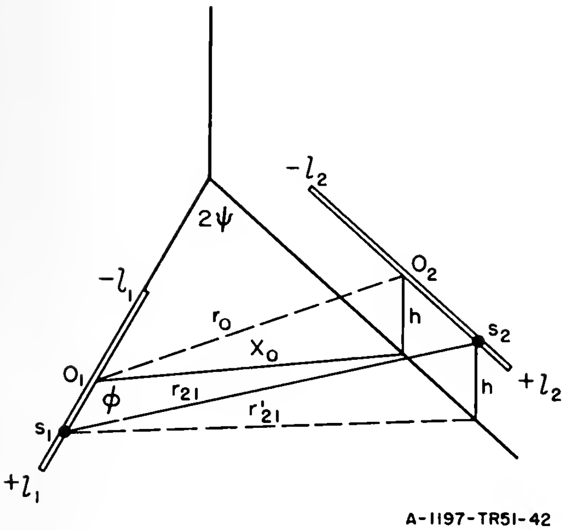

FIG. 1

SCHEMATIC DIAGRAM OF TWO SKEW ANTENNAS

Starting with these general expressions, Chaney arrives, after extensive manipulation, at an expression for the mutual impedance between two skew wire antennas, in terms of the associated sine and cosine integral functions. The resulting expression is quite lengthy, and hence has not been reproduced in this report. It may be readily shown that the formula reduces to that for the mutual impedance between the two elements of an open wire X-antenna[6] when the center-to-center spacing is reduced to zero, and to the formula for the mutual impedance of two parallel wire antennas[7] when the skew angle is equal to zero.

For the case of two coplanar, half-wavelength elements with half-wavelength center-to-center spacing, the general expression is simplified to

$$\frac{Z_{12}}{15} = \sin(kr_0\tan\psi)\Big\{Sik[r_{12}(l_1l_2) + r_0\tan\psi] - Sik[r_{12}(l_1l_2) - r_0\tan\psi]$$

$$- Sik[r_{13}(l_1l_2) + 2l_1 + r_0\tan\psi] + Sik[r_{13}(l_1l_2) - 2l_1 - r_0\tan\psi]$$

$$+ Sik[r_{14}(l_1l_2) + 2l_1 - r_0\tan\psi] - Sik[r_{14}(l_1l_2) - 2l_1 + r_0\tan\psi]$$

$$+ Sik[r_{15}(l_1l_2) + r_0\tan\psi] - Sik[r_{15}(l_1l_2) - r_0\tan\psi]\Big\}$$

$$
\begin{aligned}
&+ \cos(kr_0\tan\psi) \Big\{ Cik[r_{12}(l_1l_2) + r_0\tan\psi] + Cik[r_{12}(l_1l_2) - r_0\tan\psi] \\
&\qquad - Cik[r_{13}(l_1l_2) + 2l_1 + r_0\tan\psi] - Cik[r_{13}(l_1l_2) - 2l_1 - r_0\tan\psi] \\
&\qquad - Cik[r_{14}(l_1l_2) + 2l_1 - r\tan\psi] - Cik[r_{14}(l_1l_2) - 2l_1 + r_0\tan\psi] \\
&\qquad + Cik[r_{15}(l_1l_2) + r_0\tan\psi] + Cik[r_{15}(l_1l_2) - r_0\tan\psi] \Big\} \\
&+ \sin(kr_0\cot\psi) \Big\{ Sik[r_{12}(l_1l_2) - 2l_1 + r_0\cot\psi] - Sik[r_{12}(l_1l_2) + 2l_1 - r_0\cot\psi] \\
&\qquad - Sik[r_{13}(l_1l_2) + r_0\cot\psi] + Sik[r_{13}(l_1l_2) - r_0\cot\psi] \\
&\qquad - Sik[r_{14}(l_1l_2) + r\cot\psi] + Sik[r_{14}(l_1l_2) - r_0\cot\psi] \\
&\qquad + Sik[r_{15}(l_1l_2) + 2l_1 + r_0\cot\psi] - Sik[r_{15}(l_1l_2) - 2l_1 - r_0\cot\psi] \Big\} \\
&+ \cos(kr_0\cot\psi) \Big\{ Cik[r_{12}(l_1l_2) + 2l_1 - r_0\cot\psi] + Cik[r_{12}(l_1l_2) - 2l_1 + r_0\cot\psi] \\
&\qquad - Cik[r_{13}(l_1l_2) + r_0\cot\psi] - Cik[r_{13}(l_1l_2) - r_0\cot\psi] \\
&\qquad - Cik[r_{14}(l_1l_2) + r_0\cot\psi] - Cik[r_{14}(l_1l_2) - r\cot\psi] \\
&\qquad + Cik[r_{15}(l_1l_2) + 2l_1 + r_0\cot\psi] + Cik[r_{15}(l_1l_2) - 2l_1 - r_0\cot\psi] \Big\} \\
&+ j \Big(\sin(kr_0\tan\psi) \Big\{ Cik[r_{12}(l_1l_2) + r_0\tan\psi] - Cik[r_{12}(l_1l_2) - r_0\tan\psi] \\
&\qquad - Cik[r_{13}(l_1l_2) + 2l_1 + r_0\tan\psi] + Cik[r_{13}(l_1l_2) - 2l_1 - r_0\tan\psi] \\
&\qquad + Cik[r_{14}(l_1l_2) + 2l_1 - r_0\tan\psi] - Cik[r_{14}(l_1l_2) - 2l_1 + r_0\tan\psi] \\
&\qquad + Cik[r_{15}(l_1l_2) + r_0\tan\psi] - Cik[r_{15}(l_1l_2) - r_0\tan\psi] \Big\} \\
&- \cos(kr_0\tan\psi) \Big\{ Sik[r_{12}(l_1l_2) + r_0\tan\psi] + Sik[r_{12}(l_1l_2) - r_0\tan\psi] \\
&\qquad - Sik[r_{13}(l_1l_2) + 2l_1 + r_0\tan\psi] - Sik[r_{13}(l_1l_2) - 2l_1 - r_0\tan\psi] \\
&\qquad - Sik[r_{14}(l_1l_2) - 2l_1 + r_0\tan\psi] - Sik[r_{14}(l_1l_2) + 2l_1 - r_0\tan\psi] \\
&\qquad + Sik[r_{15}(l_1l_2) - r_0\tan\psi] + Sik[r_{15}(l_1l_2) + r_0\tan\psi] \Big\}
\end{aligned}
$$

$$
\begin{aligned}
- \sin (kr_0\cot\psi) \Big\{ & Cik[r_{12}(l_1l_2) + 2l_1 - r_0\cot\psi] - Cik[r_{12}(l_1l_2) - 2l_1 + r_0\cot\psi] \\
& + Cik[r_{13}(l_1l_2) + r_0\cot\psi] - Cik[r_{13}(l_1l_2) - r_0\cot\psi] \\
& + Cik[r_{14}(l_1l_2) + r_0\cot\psi] - Cik[r_{14}(l_1l_2) - r_0\cot\psi] \\
& - Cik[r_{15}(l_1l_2) + 2l_1 + r_0\cot\psi] + Cik[r_{15}(l_1l_2) - 2l_1 - r_0\cot\psi] \Big\} \\
- \cos (kr_0\cot\psi) \Big\{ & Sik[r_{12}(l_1l_2) - 2l_1 + r_0\cot\psi] + Sik[r_{12}(l_1l_2) + 2l_1 - r_0\cot\psi] \\
& - Sik[r_{13}(l_1l_2) + r_0\cot\psi] - Sik[r_{13}(l_1l_2) - r_0\cot\psi] \\
& - Sik[r_{14}(l_1l_2) + r_0\cot\psi] - Sik[r_{14}(l_1l_2) - r_0\cot\psi] \\
& + Sik[r_{15}(l_1l_2) + 2l_1 + r_0\cot\psi] + Sik[r_{15}(l_1l_2) - 2l_1 - r_0\cot\psi] \Big\} \Big\rangle
\end{aligned}
\tag{6}
$$

The functions $Ci(x)$ and $Si(x)$ are the usual cosine and sine integral functions defined as follows:

$$
\begin{aligned}
Ci(x) &= -\int_x^\infty \frac{\cos t}{t}\, dt \\
Si(x) &= \int_0^x \frac{\sin t}{t}\, dt \; .
\end{aligned}
\tag{7}
$$

The term r_{12} has already been defined. The terms r_{13}, r_{14}, and r_{15} were introduced as an aid to integration. They are defined in the coplanar case under consideration as

$$
\begin{aligned}
r_{12}\ (l_1l_2) &= [2l_1^2 + r_0^2 - 2l_1^2\cos 2\psi - 2l_1r_0\sin 2\psi]^{\frac{1}{2}} \\
r_{13}\ (l_1l_2) &= [2l_1^2 + r_0^2 + 2l_1^2\cos 2\psi + 2l_1r_0\sin 2\psi]^{\frac{1}{2}} \\
r_{14}\ (l_1l_2) &= [2l_1^2 + r_0^2 + 2l_1^2\cos 2\psi - 2l_1r_0\sin 2\psi]^{\frac{1}{2}} \\
r_{15}\ (l_1l_2) &= [2l_1^2 + r_0^2 - 2l_1^2\cos 2\psi + 2l_1r_0\sin 2\psi]^{\frac{1}{2}} \; .
\end{aligned}
\tag{8}
$$

It is interesting to note that these quantities, appearing as a result of integration, are the physical lengths shown in Fig. 2.

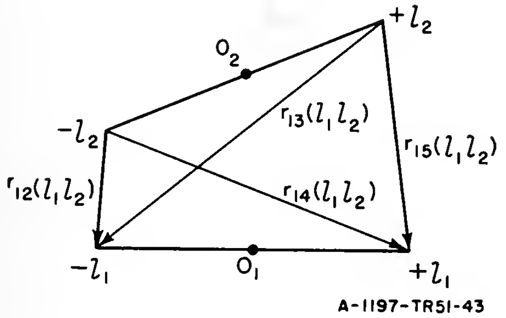

FIG. 2

DIAGRAM ILLUSTRATING THE QUANTITIES r_{12}, r_{13}, r_{14}, r_{15}

At this point it should be recalled that the general expression was derived under the hypothesis of sinusoidally distributed currents and is, therefore, most accurate for antennas in the neighborhood of a half wavelength long, and is unsatisfactory for antennas very close to one wavelength long.

B. COMPUTATION OF MUTUAL IMPEDANCE

Values of the resistive and reactive components of the mutual impedance for the case of two coplanar, half-wavelength elements with half-wavelength center-to-center spacing have been computed from Eq. (8) for skew angles between zero and 90 deg at 10-deg intervals. The results of these computations appear in Table I.

TABLE I

MUTUAL IMPEDANCE FOR VARIOUS ANGLES OF SKEW

Skew Angle 2ψ	Resistive Component R	Reactive Component X
0°	-12.5	-30.1
10°	-12.2	-29.8
20°	-11.1	-29.0
30°	-10.1	-26.4
40°	- 8.2	-23.4
50°	- 6.7	-20.4
60°	- 4.8	-15.7
70°	- 3.1	-11.2
80°	- 1.6	- 6.3
90°	0	0

CHAPTER 3

MEASUREMENT CONSIDERATIONS

A. PROBLEMS INVOLVED

In order that the experimental results might be used to test the analytical results, it was necessary to use an experimental model which conformed as nearly as possible to the assumptions made in deriving the formula.

The use of a wire antenna model would have necessitated suspension of the model in free space and the use of a reflectionless feed system, as well as balanced impedance measurements. The difficulties anticipated in obtaining accurate measurements in view of these obstacles ruled out this approach to the problem.

In order to avoid the difficulties inherent in the use of thin dipoles, the measurements described here were made by means of the slot complement of the antenna system (Fig. 3.) The method consisted of measuring the input admittance of the driven slot for the case where the parasitic slot is short-circuited at its center and that where it is open-circuited. The driven slot was fed by a small air-dielectric coaxial cable which permitted the use of slotted-line techniques for the measurements. To minimize reflections, the cable was partially embedded in the ground screen.

The network equations for a pair of slot antennas are

$$I_1 = V_1 Y_{11} + V_2 Y_{12}$$

and

$$I_2 = V_1 Y_{12} + V_2 Y_{22} \tag{9}$$

where

I_1 = the terminal current of slot 1,

I_2 = the terminal current of slot 2,

V_1 = the terminal voltage of slot 1

V_2 = the terminal voltage of slot 2,

Y_{11} = the self admittance of slot 1,

Y_{12} = the mutual admittance between slot 1 and slot 2,

Y_{22} = the self admittance of slot 2, and

$Y_{11} = \dfrac{I_1}{E_1}$ = the terminal admittance of slot 1.

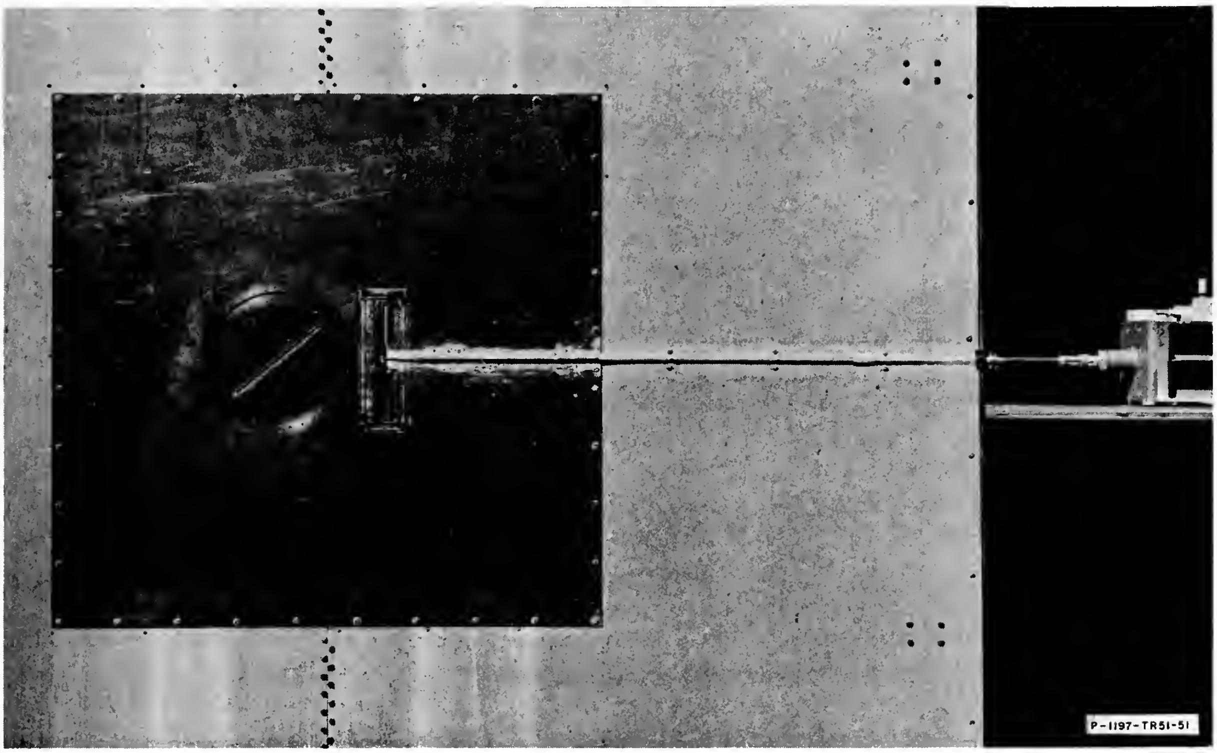

FIG. 3

GROUND PLANE

For this investigation, it was assumed that $Y_{11} = Y_{22}$; therefore, with the second slot short-circuited, the input admittance at the terminals of the first slot is

$$Y_{in} = Y_{11}$$

and, with the second slot short-circuited

$$Y_{in} = Y_{11} - \frac{Y_{12}^2}{Y_{11}} \, . \tag{10}$$

The mutual admittance Y_{12} was computed from these two expressions, using measured values of Y_{in} for the two termination conditions.

Since the mutual admittance between two slots is related to the mutual impedance of the complementary dipoles by the equation

$$Z_{12}(\text{dipole}) = \frac{\eta_0^2}{4} Y_{12}(\text{slot}), \tag{11}$$

where

$$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi \text{ ohms},$$

the desired mutual impedance between two coplanar skew antennas may be obtained with the complementary slot model. The last equation, which stems from Babinet's principle as formulated by Booker,[8] is exact if the screen is plane, perfectly conducting, and vanishingly thin.

In order to establish the range of magnitudes of the quantities to be measured, the mutual impedance between two coplanar skew wire antennas was examined for the specific case where

$$\begin{aligned} l &= \lambda/4, \\ 2\psi &= 0°, \\ \Omega &= 11, \text{ and} \\ r_0 &= \lambda/2 \, . \end{aligned}$$

The King-Middleton second order solution for the impedance of cylindrical antennas, in conjunction with the relation

$$Z\ (\text{dipole}) = \frac{\eta_0^2}{4} Y\ (\text{slot}),$$

was used to obtain the slot admittance Y_{11}. It was found that, for a thickness parameter $\Omega = 11$, which is shown in the next section to be applicable for the slot configuration used,

$$Y_{11} = (2.72)\,(10^{-3})\,\underline{|24.70°} \,.$$

Chaney's equation for the mutual impedance for the above case, when converted to a slot admittance, gives

$$Y_{12} = (0.891)\,(10^{-3})\,\underline{|-112.5°} \,.$$

From the relation

$$Y_{in} = Y_{11} - \frac{Y_{12}^2}{Y_{11}} \,,$$

it was found that

$$Y_{in} = (2.71)\,(10^{-3})\,\underline{|18.6°} \,,$$

and when normalized to the 50-ohm characteristic impedance of the slotted line,

$$y_{11} = 0.1235 + j0.0569,$$

and

$$y_{in} = 0.1285 + j0.0433 \,.$$

This computation indicated that the maximum difference to be expected in the voltage standing-wave ratio was about three-tenths in the range of VSWR between 7 and 8, and that the maximum difference in the locations of the minima for Y_{in} and Y_{11} was about 0.2 cm at a frequency of 1000 Mc. It should be noted that, theoretically, as the skew angle approaches 90 deg, the value of Y_{in} approaches the value of Y_{11} and, in the limit, Y_{in} equals Y_{11}.

The foregoing facts made it apparent that it would be necessary to establish a highly sensitive and precise slotted line technique to prevent normal experimental variations from masking the effect of the parasitic slot.

B. MEASUREMENT TECHNIQUES USED

In applying Babinet's principle it was assumed that the ground plane is infinitely large, perfectly conducting, and vanishingly thin. In the experimental setup a vertical 1/16-in. sheet-aluminum ground plane 8 ft high and 6 ft wide was used. At the measurement frequency of 1000 Mc, the closest edge of the ground plane was 2.5 wavelengths from any portion of either slot. This was considered sufficient to reduce reflections from the edges of the ground plane to negligible proportions. The center section of the ground plane in which the slots were placed consisted of a 1/16-in. copper sheet 2.5 ft square. The area around the fixed, driven slot was milled to reduce the thickness of the sheet at the slot to 0.030 in. (Fig. 4).

The rotatable parasitic slot was cut in the center of an 8-in. diameter copper disk 0.02 in. thick. There were 36 holes around the periphery of the disk, to permit its attachment to the 1/16-in. copper sheet with binder-head machine screws. When the disk was turned, these holes matched those around the circular cut-out in the ground plane for every successive 10 deg of rotation, so that it was possible to change the skew angle in 10-deg increments.

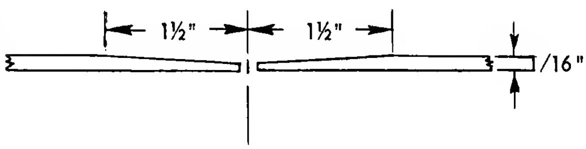

FIG. 4

MILLED AREA IN VICINITY OF DRIVEN SLOT

The driven slot was fed by a brass, 50-ohm, coaxial line having a 3/8-in. outside diameter. Styrofoam spacers were used to support the center conductor. Since Styrofoam has a dielectric constant of 1.03, the line behaved essentially as an air dielectric line from 600 to 2100 Mc. At the point of feed, the coaxial line was tapered to an outside diameter of 3/16 in. by a 2-in. electroformed taper section; at the feedpoint, the inner conductor of this 50-ohm taper had a diameter of 1/32 in. To minimize reflections from the coaxial line, the ground plane was slotted and the coaxial line inserted. The outer brass conductor was seated to the copper plate and banded to the aluminum section to preserve the continuity of the ground plane. At the edge of the ground plane, this coaxial line was connected to a Hewlett-Packard 805A Slotted Line. The experimental setup is shown in Fig. 5.

The wavelength was determined by measuring the distance between two minima on the slotted line. A plot of the position of the minima versus wavelength, with the line short-circuited, was a straight line, indicating that there was no appreciable difference between the characteristic impedance of the coaxial line and that of the slotted line, and that there were no appreciable reflections from the Styrofoam spacers in the coaxial line.

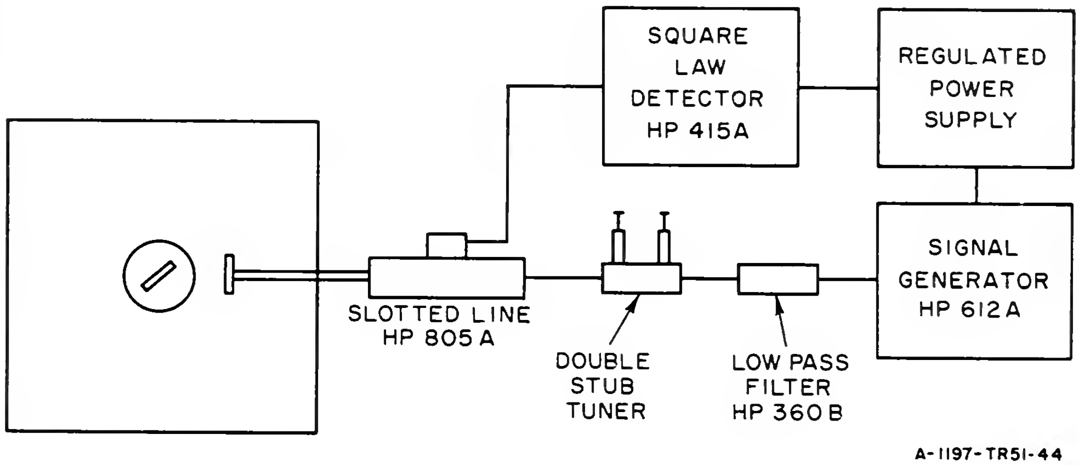

FIG. 5
BLOCK DIAGRAM OF APPARATUS

The voltage standing-wave ratio at the antenna terminal is determined from the relations described by D. D. King.[10]

$$\sinh\,(\rho + \alpha\, d\, \mathrm{min}) = \sin k \tag{12}$$

$$\mathrm{VSWR} = \coth \rho \tag{13}$$

where

$$k = \frac{2\pi}{\lambda}$$

δ = one-half the distance between the 3-db points on either side of a voltage minimum,

α = the attenuation constant of the coaxial line in nepers

d min = the distance from load to first minimum, and

ρ = the damping factor of the terminal impedance; it is equal to zero for a short circuit.

In the determination of the VSWR the meter readings were corrected in accordance with a calibration curve plotted for the crystal and amplifier. The attenuation constant, α, was obtained from Eq. (12) for the case where the line was short-circuited at the antenna terminals. To minimize the effect of experimental error, each reading necessary to determine Y_{11} or Y_{in} was taken at least five times and the values averaged.

After determining the corrected voltage standing-wave ratio, Y_{11} and Y_{in} were calculated by the formula[11]

$$Y = \frac{1}{R_c} \frac{S - j \tan kd \min}{1 - jS \tan kd \min} \qquad (14)$$

where

R_c = characteristic resistance of slotted line, and

S = VSWR.

To assess the accuracy of the foregoing techniques, measurements were made of the input impedance, $\frac{1}{Y_{in}}$, of a single rectangular slot antenna from 600 to 1200 Mc, and the results compared with the values obtained from the King-Middleton second order impedance curves for cylindrical antennas transformed to slot impedances by application of Babinet's principle. For the slot antenna considered (Fig.6), $l/D = 60$. Since the diameter of a complementary cylindrical wire antenna is equivalent to one half the diameter of the rectangular slot antenna,[12]

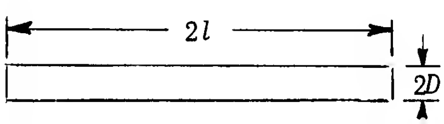

FIG. 6

SCHEMATIC OF SLOT

$$\Omega = 2 \ln \frac{4l}{D} = 10.96 .$$

A comparison of the theoretical results for $\Omega = 11$ with measured values (Fig. 7) indicates good agreement.

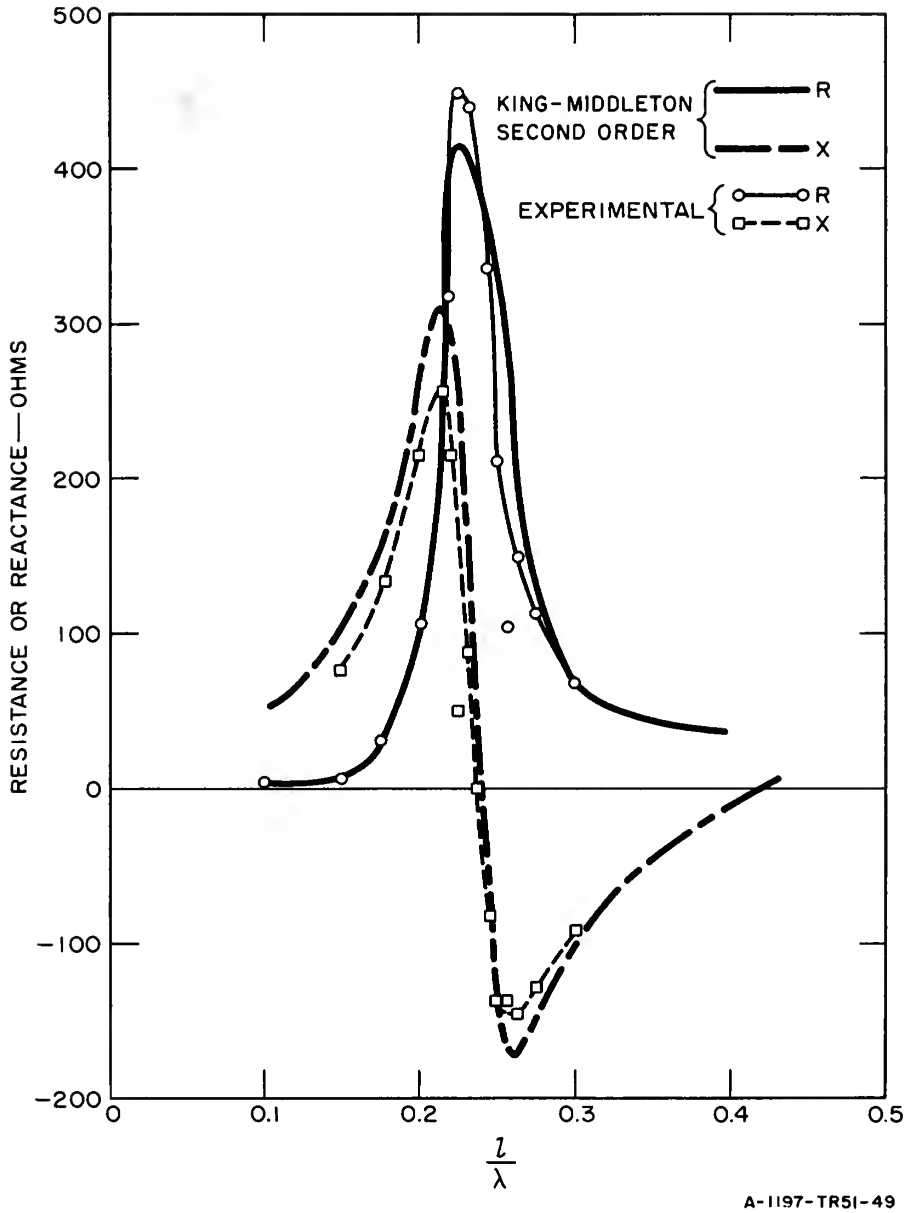

FIG. 7

COMPARISON OF THEORETICAL AND MEASURED INPUT IMPEDANCE FOR A RECTANGULAR SLOT ANTENNA

C. RESULTS

The input admittance and Y_{11} of the two-slot array for various angles of skew were measured. The variations of the values as a function of skew angle are shown in Figs. 8 and 9. Because Y_{11} varied only slightly with the angle of skew,* the value of Y_{11} for a skew angle of 90 deg was used in computing Y_{12}.

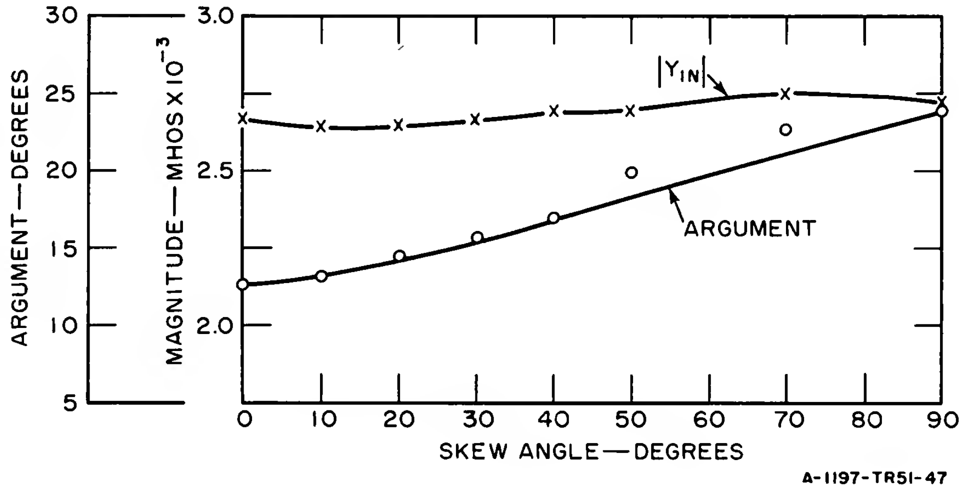

FIG. 8
MEASURED VALUES OF Y_{in} FOR
VARIOUS ANGLES OF SKEW

A plot of the experimental and theoretical values of the mutual impedance between two coplanar, half-wave antennas with half-wave center-to-center spacing is given in Fig. 10. The general shape of the impedance curves predicted by theory agrees well with the shape of those determined by measurement. However, the experimental curves are about 30% greater in value than those predicted by first order theory for both the R_{12} and X_{12} components.

If the condition of zero skew angle is considered, other experimental and theoretical results are available for comparison.[9, 13, 14] The

---

* Note that Y_{11} is defined as the input admittance of the driven slot with the parasitic slot present but short-circuited at its terminals. The shorted parasitic will have a small effect on the input admittance of the driven slot and hence on Y_{11}. This effect will depend upon the skew angle.

values of the mutual impedance between two parallel half-wave elements with half-wave center-to-center spacing as measured by Blasi[13] and by Moritz[14] together with King's first and approximate second order solutions are tabulated in Table II.

The measurement techniques employed by Blasi and Moritz both differed from those used in this investigation.

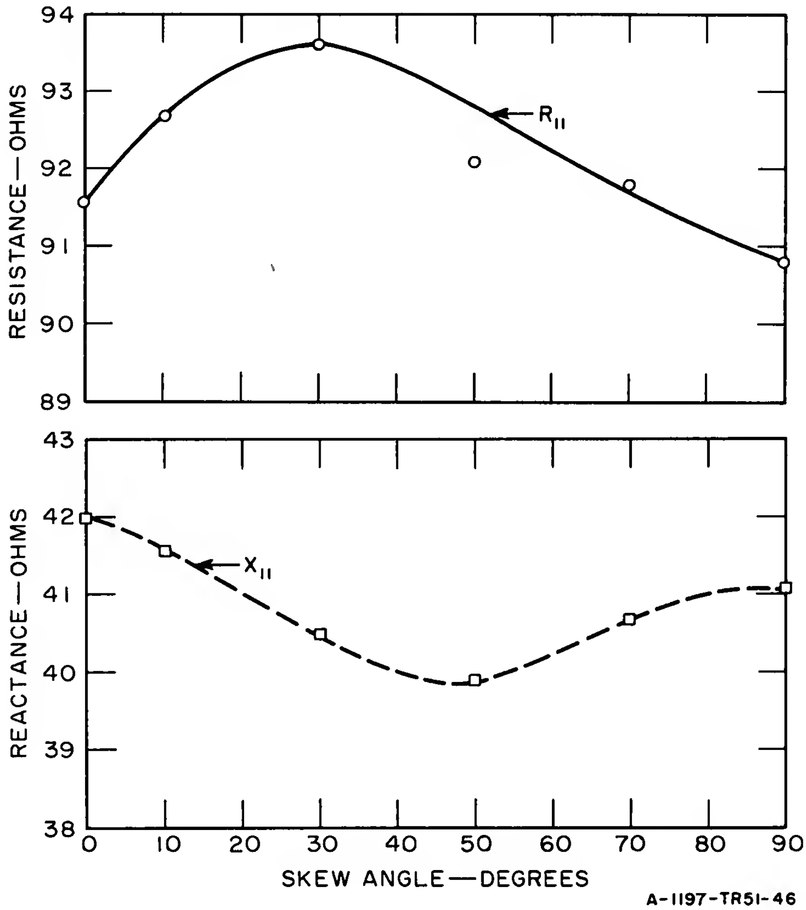

FIG. 9

MEASURED VALUES OF Z_{11} FOR TWO COPLANAR, HALF-WAVE ANTENNAS WITH HALF-WAVE CENTER-TO-CENTER SPACING FOR VARIOUS ANGLES OF SKEW

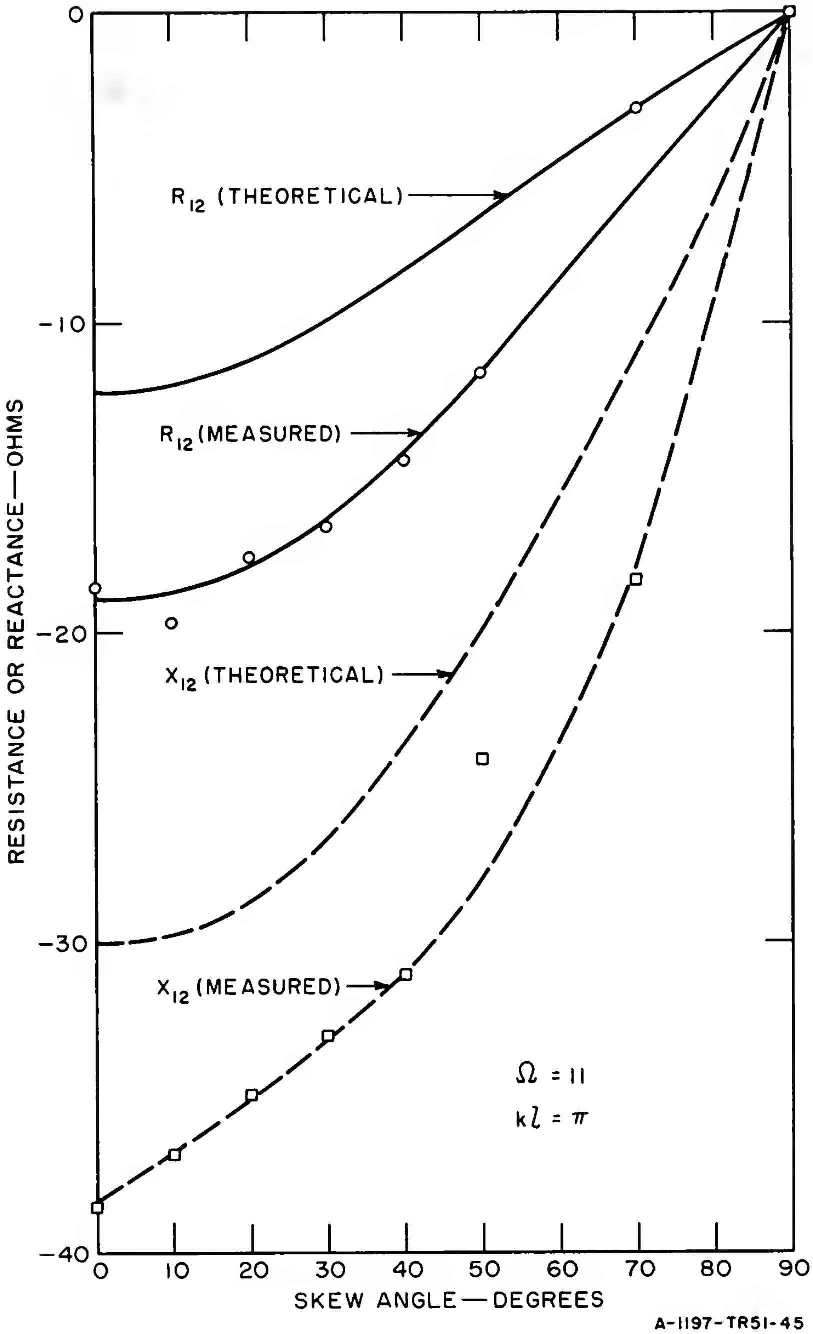

FIG. 10

THE MUTUAL IMPEDANCE BETWEEN TWO COPLANAR, HALF-WAVE ANTENNAS WITH HALF-WAVE CENTER-TO-CENTER SPACING FOR VARIOUS ANGLES OF SKEW

TABLE II

COMPARISON OF THEORETICAL AND MEASURED VALUES OF MUTUAL IMPEDANCE

R	X	Origin
-12.5	-30.1	King first order theory, Ω = 11
-15.2	-30.4	King approximate second order theory, Ω = 10
-15.2*	-30.4*	Blasi measured, Ω = 11
-28.5*	-34*	Moritz measured, Ω = 9.3
-18.6	-38.6	This investigation, Ω = 11

*These values were taken from curves.

Additional data were obtained to indicate the variation of the mutual impedance with variation in frequency, between two fixed-length antennas with fixed center-to-center spacing and fixed skew angle. These data are presented in Fig. 11 and Fig. 12.

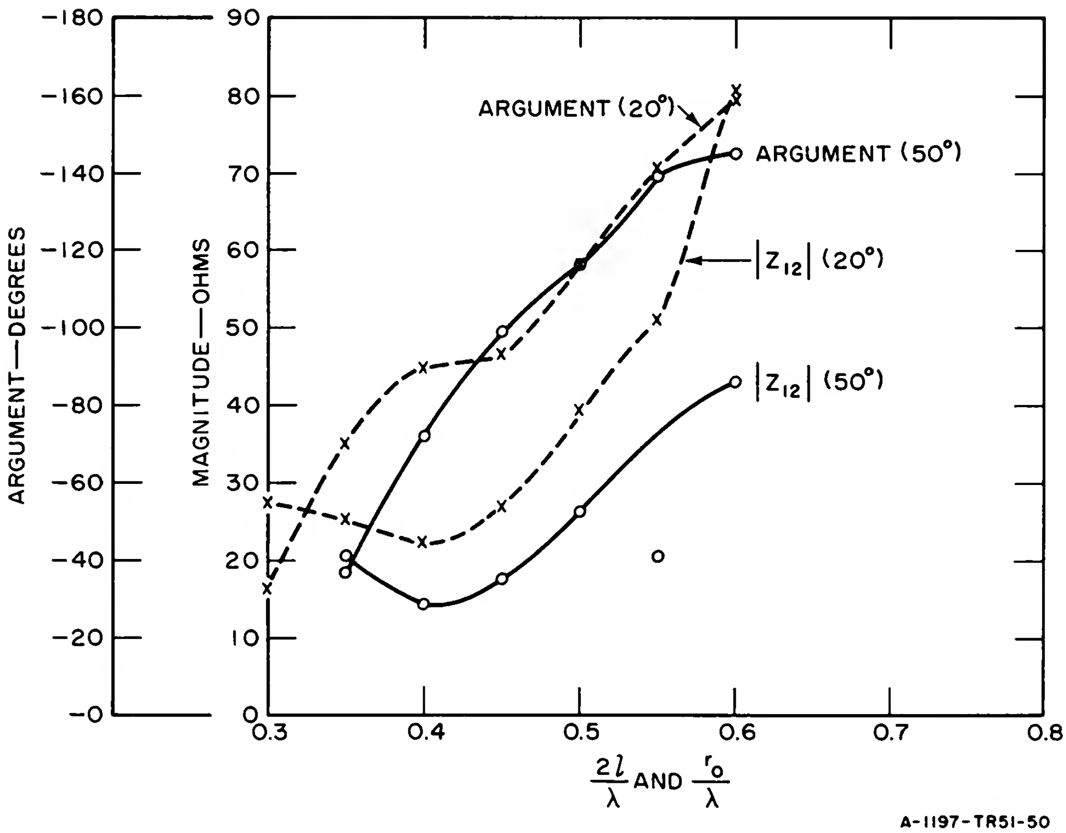

FIG. 11

MEASURED VALUES OF $|Z_{12}|$ BETWEEN TWO FIXED-LENGTH ANTENNAS WITH FIXED CENTER-TO-CENTER SPACING AND FIXED SKEW ANGLE

D. ALTERNATIVE METHODS OF APPROACH

Because of the difficulty encountered in this investigation in measuring the small changes in Y_{in}, other approaches to the measurement problem may be desirable. Blasi, in his paper "The Theory and Application

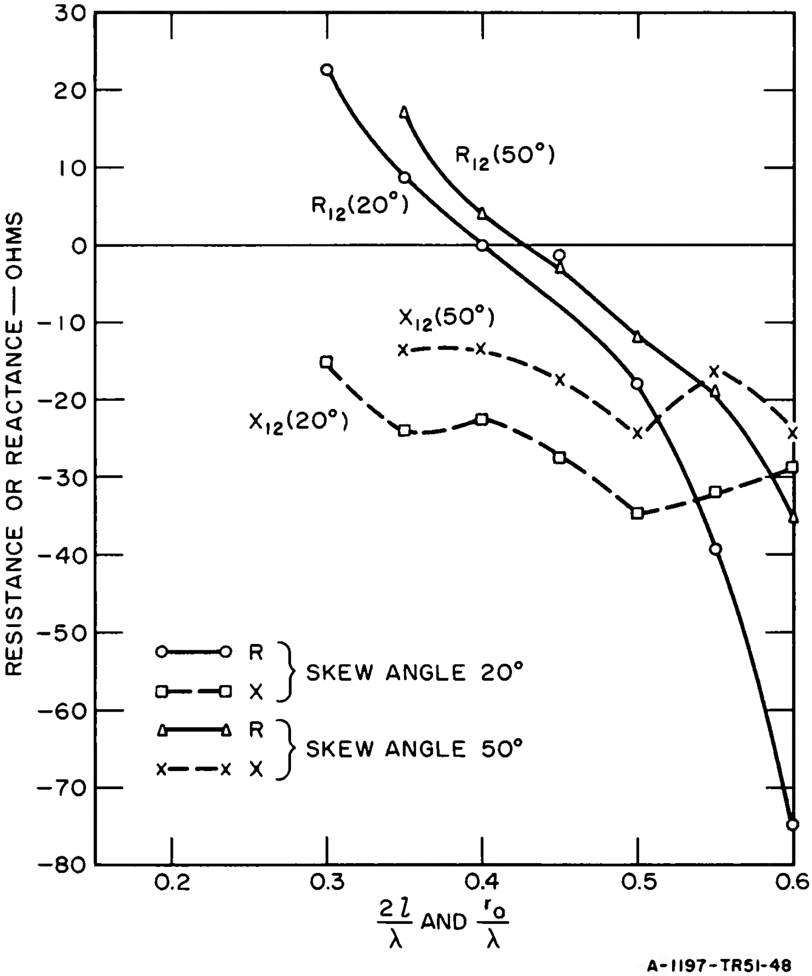

FIG. 12

MEASURED VALUES OF Z_{12} BETWEEN TWO FIXED-LENGTH ANTENNAS WITH FIXED CENTER-TO-CENTER SPACING AND FIXED SKEW ANGLE

of the Mutual Coupling Factor,"[13] introduced a measurement technique that could be applied to the skew antenna problem by a slot array. His technique is based on the relation

$$Z_{12} = M_r \sqrt{Z_{11}Z_{22}} \tag{15}$$

where

$$M_r \quad = \quad \text{radiation mutual coupling factor.}$$

The procedure is to adjust a triple stub tuner placed between the slotted line and the drive element to give a VSWR as near unity as possible with the driven element isolated, and then introduce the parasitic element which will disturb the initial matched condition. A quantity Z'_{12} can now be measured. Blasi shows that

$$M_r = \frac{Z'_{12}}{R_c} . \tag{16}$$

Therefore, by assuming $Z_{11} = Z_{22}$, and by using either a measured value of Z_{11} or the King-Middleton second order solution for the self impedance of a cylinderical antenna, the mutual impedance may be calculated.

A second possible approach would be to use a ground plane array of monopoles as shown in Fig. 13. If the image of antenna 2 is considered to be the lower half of antenna 3, and the image of antenna 3 to be the lower half of antenna 2, then the network equations for this array are

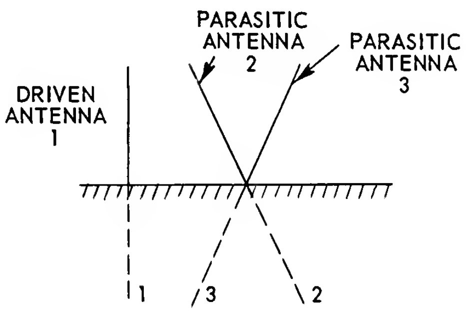

FIG. 13

SCHEMATIC DIAGRAM OF MONOPOLE TECHNIQUE

$$\begin{aligned} V_1 &= I_1 Z_{11} + I_2 Z_{12} + I_2 Z_{12} \\ 0 &= I_1 Z_{12} + I_2 Z_{11} + I_2 Z_{23} \\ 0 &= I_1 Z_{12} + I_2 Z_{23} + I_2 Z_{11}, \end{aligned} \tag{17}$$

where it is assumed that

$$\begin{aligned} Z_{11} &= Z_{22} = Z_{33} \\ Z_{12} &= Z_{13} \\ I_2 &= I_3 \end{aligned} \tag{18}$$

The input impedance is thus given by

$$Z_{in} = Z_{11} - 2\frac{Z_{12}^2}{Z_{11} + Z_{23}}. \tag{19}$$

From this relation the mutual impedance may be calculated, using either measured or computed values for Z_{11} and Z_{23}.

Use of a ground plane array of monopoles would permit the measurement of cases other than the coplanar case.

CHAPTER 4

SUMMARY AND CONCLUSIONS

It has been demonstrated that there is good correlation between the general shape of the theoretical curves for the mutual impedance between two coplanar skew wire antennas computed from Chaney's general formula and the curves determined by measurement. There is, however, a difference in magnitude of about 30% between the theoretical and measured values.

A comparison with other available results for the specific case of two parallel half-wave elements for a half-wave center-to-center spacing shows wide dispersion of results from different experimentors.

To overcome some of the difficulties encountered in this investigation, several alternative measuring techniques were briefly discussed.

TABLE OF SYMBOLS AND ABBREVIATIONS

SYMBOL	QUANTITY
a	Dependent variable, $a = kh \tan \psi$
b	Dependent variable, $b = kh \cot \psi$
$Ci(x)$	Cosine integral
$Cia(x)$, $Cib(x)$	Associated cosine integral
D	Half width of slot
d_{min}	Distance from load to first minimum
$e(r_{12})$	Simplifying factor, $e(r_{12}) = \dfrac{e^{-jkr_{12}}}{r_{12}}$
$f_i(P_i)$	Current distribution on wire No. i
h	Shortest distance between the lines of the two antenna axes
I	Terminal current
j	Unit imaginary number in complex plane
k	Wavelength constant, $k = \dfrac{2\pi}{\lambda}$
l	Half length of antenna
M_r	Radiation mutual coupling factor
P_i	Any point along wire No. i
r_{12}	Distance from point one to point two
R_c	Characteristic resistance
R_e	Real part of
r_0	Center to center distance between two antennas
S	Voltage standing wave ratio
$Si(x)$	Sine integral
$Sia(x)$, $Sib(x)$	Associated sine integral
t, x	Independent variables
V	Terminal voltage

SYMBOL	QUANTITY
vhf	Very high frequency, 30 to 300 Mc
VSWR	Voltage standing wave ratio
x_0	Projection of the line of centers upon the plane through antenna 1 parallel to antenna 2
Y	Admittance
y	Normalized admittance
Z	Impedance
α	Attenuation constant
δ	One half the distance between +3-db points on either side of a voltage minimum
ϵ_0	Permitivity of free space $(36\pi \times 10^9)^{-1}$ farads per meter
η_0	Intrinsic impedance of free space, $\eta_0 = 120\pi$
λ	Wavelength
μ_0	Permeability of free space ($4\pi \times 10^{-7}$ henries per meter)
ρ	Damping factor which can be determined from the width of the minimum
φ	Angle between x_0 and the positive sense of antenna 1
2ψ	Vertex angle between positive senses of antenna 1 and the coplanar line parallel with antenna 2
ω	Angular frequency
Ω	Hallen's constant, $\Omega = 2ln\frac{2l}{D}$
$\Diamond_1$	Operator "deltil," with subscript indicating the position at which the differentions are to be performed, $\Diamond_1 = \nabla_1 (\nabla_1 \cdot\ ) + k^2$
*	Complex conjugate

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